TRIGONOMETRY

Identities

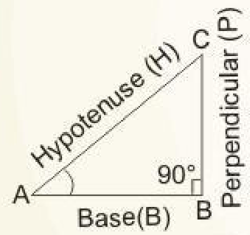
$\frac{AB^2}{AC^2}+\frac{BC^2}{AC^2}=1 = \sin^2 A + \cos^2 A = 1$ ← AC^2

$\frac{AB^2}{BC^2}+1=\frac{AC^2}{BC^2} = \cot^2 A+1 = \text{cosec}^2 A$ ← BC^2

$1+\frac{BC^2}{AB^2}=\frac{AC^2}{AB^2} = 1+\tan^2 A = \sec^2 A$ ← AB^2

Divide both sides by

$AB^2 + BC^2 = AC^2$



Complementary Angles

$\sin(90-A)=\frac{AB}{AC}$	$\tan(90-A)=\frac{AB}{BC}$	$\text{cosec}(90-A)=\frac{AC}{AB}$
$\cos A=\frac{AB}{AC}$	$\cot A=\frac{AB}{BC}$	$\sec A=\frac{AC}{AB}$
$\cos(90-A)=\frac{BC}{AC}$	$\cot(90-A)=\frac{BC}{AB}$	$\sec(90-A)=\frac{AC}{BC}$
$\sin A=\frac{BC}{AC}$	$\tan A=\frac{BC}{AB}$	$\text{cosec} A=\frac{AC}{BC}$

e.g. If $\sin 3\theta = \cos(\theta - 6°)$ and 3θ and $\theta - 6°$ are acute, find the value of θ.

Sol. $\sin 3\theta = \cos(\theta - 6°)$

$\Rightarrow \cos(90° - 3\theta) = \cos(\theta - 6°)$

$\Rightarrow 90° - 3\theta = \theta - 6°$

$\Rightarrow 4\theta = 96°$

$\Rightarrow \theta = 24°$.

e.g. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then Prove that : $x^2 + y^2 + z^2 = r^2$.

Sol. $x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\phi$

$x^2 + y^2 + z^2$

$= r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

$= r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + r^2\cos^2\theta$

$= r^2\sin^2\theta + r^2\cos^2\theta$

$= r^2(\sin^2\theta + \cos^2\theta)$

$= r^2$

T-ratios

$\sin A = \frac{BC}{AC} = \frac{P}{H}$ $\text{cosec} A = \frac{AC}{BC} = \frac{H}{P}$

$\cos A = \frac{AB}{AC} = \frac{B}{H}$ $\sec A = \frac{AC}{AB} = \frac{H}{B}$

$\tan A = \frac{BC}{AB} = \frac{P}{B}$ $\cot A = \frac{AB}{BC} = \frac{B}{P}$

Interrelationship between T-ratios

$\sin A = \frac{1}{\text{cosec} A}$ $\cos A = \frac{1}{\sec A}$ $\tan A = \frac{1}{\cot A}$

Trigonometric Ratios of Some Specific Angles

θ / T-ratios	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Simplified trigonometric values

NCERT / X / Trigonometry